Studies on the Mechanism of Draw Resonance in Melt Spinning

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SYNOPSIS

The physical mechanism of draw resonance of polymer fluids in melt spinning has been studied. It is proposed, according to the cross-sectional area dependence of the local draw ratio of the filament along the spinline, that filament drawing can be divided into three modes which are given in the main text. In particular, it is found that the nonuniformizing draw mode is the necessary condition of draw resonance. The mechanism is certified through the analytical solution of isothermal melt spinning under uniform spinning stress and the critical draw ratio of isothermal and uniform tension melt spinning. The mechanism was employed to analyze the promotive and suppressive factors of draw resonance in a real spinning system and the development of filament unevenness along the spinline.

INTRODUCTION

Draw resonance is a term describing the phenomenon that a cyclic variation of filament diameter or film thickness occurs in melt spinning or film-forming processes despite maintaining constant extrusion rate and the take-up velocity. This phenomenon appears to have been first reported in the work of Christensen[1] and Miller[2] in the early 1960s. Draw resonance is an undesirable behavior of polymer fluids which affects the stability of polymer processing and causes fluctuation of filament diameter or film thickness. For this reason, much attention has been paid by workers in the polymer rheology and processing fields and has been the subject of many investigations in the past three decades.[3-8]

The "heat effect" is one theoretical interpretation for the mechanism of draw resonance.[9] The explanation of the heat effect may be summarized as follows. When a polymer melt is extruded through a spinneret hole, work done by the internal friction of molecules and by the friction between the wall of the spinneret hole and polymer melt will be converted into heat. This heat generation tends to decrease the viscosity of the extruded section of the filament. The filament section will be additionally deformed to such a high extent that the above sections will be stretched to a smaller extent. The thinner section will be cooled down at a faster rate due to its smaller heat capacity. This works to increase the viscosity in the thinning section. As a consequence, further thinning of the section is stopped and a new cycle of thinning will be started at a section just extruded. The cycle repetition of the thinner and thicker sections of filament is explained to develop in this manner. It is difficult, however, to interpret data using the heat effect theory due to the experimental and theoretical facts that draw resonance can be observed in isothermal melt spinning where there is no heat removal from the filament to the surrounding air.

Another rheological explanation of draw resonance has been proposed by Han and Kim.[10] They have attributed the onset of draw resonance to the elasticity of polymer fluids, based on the facts that the critical draw ratio for draw resonance increases, and die swell decreases with increasing temperature of the polymer melt. However, it has been reported by Fisher and Denn,[11] Pearson and co-workers,[12,13] Kase and co-workers,[14,15] and Toriumi and Konda[16] that the critical draw ratio for Newtonian fluids is

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approximately 20 in isothermal spinning. The explanation, unfortunately, does not explain the fact that Newtonian fluids can show the phenomenon of draw resonance.

For studying the mechanism of draw resonance it is desirable to find the analytical solution of the governing equations of melt spinning. Unfortunately, however, the governing equations are simultaneous nonlinear partial differential and are as yet unsolved. The numerical simulation and eigenvalue analysis for the linearized governing equations have been carried out for finding the critical conditions of draw resonance. These numerical methods are helpful especially in the study of the critical conditions of melt spinning, the wave form, and the period of draw resonance.

It is the purpose of this paper to present a theoretical study for the mechanism of draw resonance in melt spinning. Based on the proposed mechanism, a simple method to calculate the critical draw ratio of draw resonance was developed. Furthermore, the mechanism was employed to analyze the developing manner of filament unevenness along the spinline. All the theoretical and experimental results show excellent agreement with the proposed mechanism.

THE MECHANISM OF DRAW RESONANCE

Definition of the Draw Mode

Before the discussion of the mechanism of draw resonance, the draw mode of the filament in melt spinning is considered. Figure 1 illustrates the parameters in melt spinning in Eulerian coordinates, where A_0, V_0, and T_0 are the cross-sectional area, extrusion rate, and temperature of the filament at the spinneret, respectively. A, v, T, and F denote, respectively, the cross-sectional area, velocity, temperature, and spinning tension of filament at a distance x from the spinneret. The symbol L represents the spinning length and V_w, the take-up velocity. The ratio of the take-up velocity to the extrusion rate is usually known as the draw ratio. In general, the draw ratio of the filament changes with position along the spinline. Consider a small volume, $\Delta x \pi r^2$, at a distance x from the spinneret. The deformation of the volume, $\Delta x \pi r^2$, at a time interval dt may be represented as,

$$d(\Delta x) = \frac{\partial v}{\partial x} \Delta x \, dt \qquad (1)$$

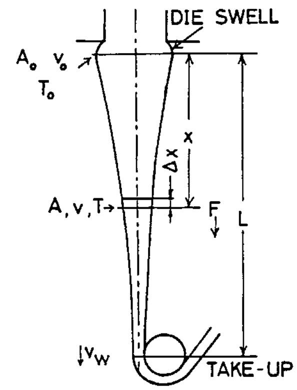

Figure 1 The parameters in melt spinning.

Therefore, the local draw ratio of the small volume, $\Delta x \pi r^2$, at the time interval dt can be written as

$$\text{local draw ratio} = 1 + \frac{d(\Delta x)}{\Delta x} = 1 + \frac{\partial v}{\partial x} dt \qquad (2)$$

Partial differentiation of eq. (2) with respect to the cross-sectional area A shows the area dependence of the local draw ratio along spinline:

$$\frac{\partial(\text{local draw ratio})}{\partial A} = \frac{\partial}{\partial A}\left(\frac{\partial v}{\partial x}\right) dt \qquad (3)$$

Since the time interval dt in eq. (2) is considered to be always positive, the parameter $(\partial/\partial A)(\partial v/\partial x)$ can be defined as the local draw mode. This parameter indicates three characteristic draw modes according to the following cases;

$$\frac{\partial}{\partial A}\left(\frac{\partial v}{\partial x}\right) \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \qquad (4)$$

The condition that $(\partial/\partial A)(\partial v/\partial x) > 0$ means a larger cross area section of filament will be stretched to a larger extent than that having smaller cross-sectional area. In this context, this draw mode could be stated as an "uniformizing draw mode." When the condition that $(\partial/\partial A)(\partial v/\partial x) = 0$ is satisfied at all the sections along spinline, all the sections of filament will be stretched to the same extent indicating the filament deforms uniformly. This draw mode is described, therefore, as an "uniform draw mode." If $(\partial/\partial A)(\partial v/\partial x) < 0$, the smaller the cross-sectional area of filament, the larger the extent the section will be stretched to. This draw mode is defined as a "nonuniformizing draw mode."

Relationship between the Draw Mode and Draw Resonance

It is explained below how draw resonance is connected with the "draw mode." Now, consider two different small volumes along the spinline in Figure 2. After a time dt, the small volume *abcd* moves to $a'b'c'd'$ and the small volume *efgh* moves to $e'f'g'h'$. According to eq. (4) one knows whether the filament is uniformized or nonuniformized from a Lagrangian point of view. From the Eulerian viewpoint, however, it is easily seen that when *abcd* reaches the position of *efgh*, the small volume has the same velocity as the small volume *efgh* had in steady state. So the draw mode only describes the attenuation behavior of the filament.

However, when the steady state of the spinline is converted into a new state, for example, the take-up velocity has a step increment, the draw mode describes how the velocity changes along the spinline in relation to the step increment. In the case of the nonuniformizing draw mode, the thinner sections near the take-up bobbin will be stretched to a larger extent to meet the take-up velocity increment, these sections become thinner and thinner. This positive feedback effect makes filament attenuation concentrate on the side of the take-up bobbin. If the take-up quantity is less than the extrusion quantity, the polymer fluid will accumulate on the side of the spinneret. The accumulated polymer fluid falls down at an identical velocity as that before the take-up velocity increment and reaches the new take-up velocity at the moment when it contacts with the take-up bobbin. After this moment the onset of the attenuation of the accumulated polymer starts, and accumulation of the fluid on the side of spinneret repeats again. Draw resonance occurs just because the process reiterates.

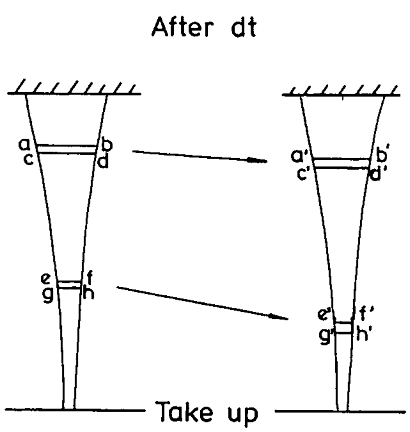

Figure 2 The draw mode is a parameter describing attenuation behavior. The ratio of the extension rates of *abcd* and *efgh* is decided by the draw mode, but when *abcd* reaches the position of *efgh* in steady state, it behaves as *efgh* did.

Draw Resonance in Isothermal Melt Spinning Under Uniform Spinning Tension

In the case of isothermal and uniform tension melt spinning for power-law polymer fluids, the governing equations are expressed as

$$\frac{\partial v}{\partial x} = \frac{F}{A\beta} \qquad (5)$$

$$\beta = \beta_0 \left(\frac{\partial v}{\partial x} \right)^{p-1} \qquad (6)$$

$$\frac{\partial A}{\partial t} + \frac{\partial (Av)}{\partial x} = 0 \qquad (7)$$

where F is the spinning tension and is constant along the spinline by the definition of uniform tension spinning. β is the Trouton viscosity, β_0 a viscosity constant and p a parameter expressing the strain rate dependence of the Trouton viscosity.

From eq. (5) and eq. (6), one obtains

$$\frac{\partial v}{\partial x} = \left(\frac{F}{\beta_0} \right)^{1/p} \frac{1}{A^{1/p}} \qquad (8)$$

Since F and β_0 are positive constants, the draw mode parameter becomes

$$\frac{\partial}{\partial A} \left(\frac{\partial v}{\partial x} \right) = -\frac{1}{p} \left(\frac{F}{\beta_0} \right)^{1/p} \frac{1}{A^{1/p+1}} < 0 \qquad (9)$$

This indicates a case of the nonuniformizing draw mode. It is clear from eq. (9) or eq. (8) that the thinner the section of filament, the larger the extent the section will be stretched to, and the smaller the power index p, the stronger is the nonuniformizing characteristic.

Figure 3 illustrates a thinking experiment which demonstrates how draw resonance occurs in melt spinning. Assuming the take-up velocity is given a step increment at time t_0 and the value p is small enough, Section 1 of the filament will have the largest velocity increment because the cross-sectional area of Section 1 is smaller than that of Section 2. At a next time, t_1, the part of Section 1, which has not yet reached the take-up bobbin, will be stretched

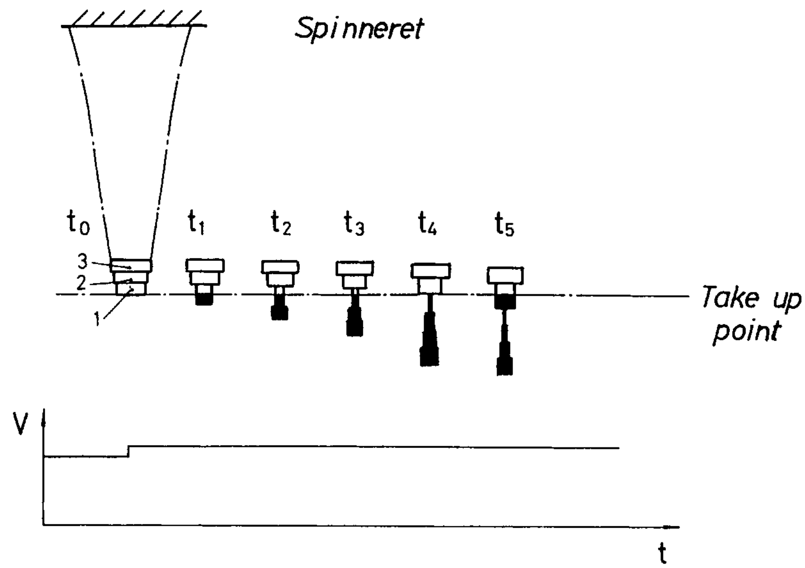

Figure 3 A schematic explanation about draw resonance. Due to the characteristic of the nonuniformizing draw mode, the section with the smallest cross-sectional area supplies the largest velocity increment. Section 2 touches the take-up bobbin with the old take-up velocity and has an abrupt increment to meet the new take-up velocity. Below the take-up point is the taken-up filament.

to a larger extent since this part is already much thinner than that at time t_0. On the other hand, Section 2 of filament will be extended little during the time interval $t_1 - t_0$, and it falls down with a velocity similar to that at time t_0. Eventually, Section 2 contacts with the take-up bobbin at time t_5 and so Section 2 will be extended abruptly at time t_5 and then behaves just like Section 1 did. Only a small part at the top of Section 2 retains its "thick" diameter, the other parts of the section become thinner. Draw resonance occurs just when the process reiterates. It is easy to imagine, therefore, that the cross-sectional area of the filament decreases monotonously along the spinline even in the case of draw resonance.

ISOTHERMAL MELT SPINNING UNDER UNIFORM STRESS FOR POWER LAW POLYMER FLUIDS

The previous section explained the mechanism of draw resonance in melt spinning schematically. In order to certify further the proposed mechanism mathematically, consider melt spinning with an uniform draw mode $\{(\partial/\partial A)(\partial v/\partial x) = 0\}$. With the uniform draw mode, all the sections of filament will be stretched to the same extent irrespective of their cross-sectional areas.

Isothermal melt spinning under uniform stress for power-law polymer fluids is an instant of uniform draw mode spinning. If the proposed mechanism is correct, it will be anticipated, that the draw resonance phenomenon will not occur in such a spinning system.

With the isothermal melt spinning under uniform spinning stress, the velocity gradient of the filament is a constant. Hence, by rewriting eq. (8), one obtains

$$\frac{\partial v}{\partial x} = \left(\frac{F}{A}\right)^{1/p} \frac{1}{\beta_0^{1/p}} = f(t) \tag{10}$$

The simultaneous governing relations, given by eq. (10) and eq. (7), are soluble analytically since the velocity of filament can be first solved from eq. (10) in this case with the boundary condition as

$$v = V_w \qquad \text{at } x = L \tag{11}$$

Integrating eq. (10) leads to,

$$v = (V_w - V_0)\frac{x}{L} + V_0 \tag{12}$$

The cross-sectional area is found by separating the independent variables time, t, and distance, x, and one can write

$$A(t, x) = A_t(t)A_x(x) \tag{13}$$

Substituting eq. (12) and eq. (13) into eq. (7) leads to,

$$\frac{A_t'}{A_t} + \frac{V_w - V_0}{L} + \left[\left(\frac{V_w - V_0}{L}\right)x + V_0\right]\frac{A_x'}{A_x} = 0 \tag{14}$$

In eq. (14), the first term is a function of time, t, the second term a constant, and the third term a function of distance, x. Hence, the first term and the third term have to be constants:

$$A_t(t) = \exp(kt) \tag{15}$$

$$A_x = C\left[\frac{V_w - V_0}{L}x + V_0\right]^{-k(L/V_w - V_0)+1} \tag{16}$$

where C and k are constants which are determined from the initial boundary conditions.

If the constant k were positive, all the sections of spinline would become infinity thick with time; on the other hand, if the constant k were negative, the spinline would disappear with increasing time. Therefore, $k = 0$ is the unique solution. This suggests that there is no unsteady solution for this spinning system when all the spinning conditions are constant.

At $x = L$, $A = A_0V_0/V_w$ and one obtains $C = A_0V_0$. The cross-sectional area $A(x, t)$ can be written as,

$$A(x, t) = A(x) = \frac{A_0V_0L}{(V_w - V_0)x + V_0L} \tag{17}$$

Equations (12) and (17) are the analytical solutions of the isothermal melt spinning under uniform stress for power-law polymer fluids. It is clear from the two equations that whenever the take-up velocity and the extrusion rate are maintained constant, no cross area variation in the taken up fiber will be observed. This in turn, indicates that the nonuniformizing draw mode is the necessary condition of draw resonance.

CRITICAL DRAW RATIOS IN ISOTHERMAL AND UNIFORM TENSION MELT SPINNING FOR POWER-LAW POLYMER FLUIDS

Equation (2) expresses the mean local draw ratio of the small volume, $\Delta x\pi r^2$, during time dt. Hence, by multiplying the cross-sectional area of the small volume with the local draw ratio, and integrating the resultant equation along the spinning direction, one obtains the draw mode of the whole spinline, S_p. It may be considered as a stability parameter of the spinline because draw resonance is a behavior of the whole spinline rather than one point or one zone and is given by,

$$S_p = \int_0^L A\frac{\partial}{\partial A}\left(\frac{\partial v}{\partial x}\right)\frac{dx}{v} \tag{18}$$

The steady state solution for the isothermal and uniform tension melt spinning for power-law polymer fluids are,[11]

$$v = V_0\left[(\psi_w^{1-q} - 1)\frac{x}{L} + 1\right]^{1/1-q} \tag{19}$$

$$A = A_0\left[(\psi_w^{1-q} - 1)\frac{x}{L} + 1\right]^{-1/1-q} \tag{20}$$

where, $\psi_w = V_w/V_0$, and is known as the draw ratio, and $q = 1/p$. For Newtonian polymer fluids, the steady state solution is written as,[14]

$$A = A_0\exp\left[-\ln(\psi_w)\frac{x}{L}\right] \tag{21}$$

$$v = V_0\exp\left[\ln(\psi_w)\frac{x}{L}\right] \tag{22}$$

Substituting eq. (19), eq. (20) or eq. (21) and eq. (22) into eq. (18), one obtains the iso-stable condition as,

$$S_p = -\frac{\ln\psi_W}{p} \tag{23}$$

The minus sign in eq. (23) shows the nonuniformizing draw mode, and the parameters on the right-hand side of eq. (23) indicate that the instability is proportional to the logarithm of the draw ratio and inversely proportional to the power-law index, p, of the fluid. By substituting the critical draw ratio 20.218 for Newtonian fluids ($p = 1$) into eq. (23),[17] one obtains the relationship between the critical draw ratio and the power-law index p as

$$\ln(\psi_W)_{cr} = 3.01p \tag{24}$$

In Figure 4 the calculated line of eq. (24) is compared with the results reported by Pearson and

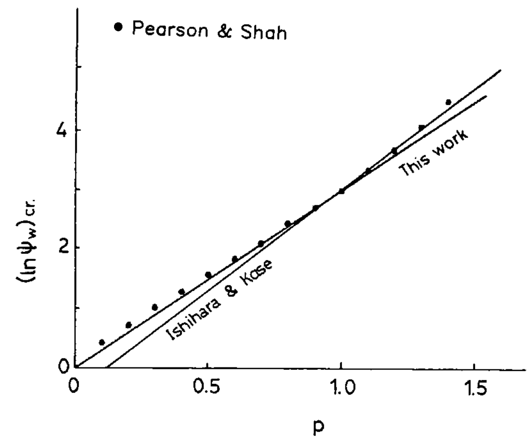

Figure 4 The logarithm of the critical draw ratio versus power-law index in isothermal and uniform tension melt spinning. Note that for an infinitely shear thinning fluid ($p = 0$), the critical draw ratio is unity.

Shah[13] who used the eigenvalue method and with the results of Ishihara and Kase[18] who used numerical simulation. The results obtained with the three methods are similar. Therefore, the draw mode parameter S_p may be considered as a useful parameter for expressing stable or unstable spinline operation.

APPLICATIONS

Promotive and Suppressive Factors of Draw Resonance in Real Melt Spinning

The mechanism of draw resonance can be used to analyze real melt spinning. Although the effects of cooling, molecular orientation, and crystallization of polymer fluids as well as the effect of spinning tension distribution along the spinline will inevitably play an important role to draw resonance, these effects have not yet been considered. The present draw mode analysis method is able to give a simple answer. In Figure 5, the main dominant factors for the draw modes are displayed. The draw mode depends on the distribution of the spinning tension and the properties of the filament. When a weaker tension exists in a thinner section, or the thinner section is stronger than the thicker one for the reason of molecular structure formation, the uniformizing draw mode will be expected and draw resonance is suppressed. Draw resonance is promoted when the inverse is true. However, the properties of the filament will be controlled by molecular orientation, crystallization, temperature, and strain rate dependencies of the Trouton viscosity. Inclusion of the above properties are too complex for the numerical or eigenvalue methods. Here, the author wants to mention that even in the isothermal and uniform tension melt spinning, molecular orientation and crystallization of polymer fluids will still make their contributions to the Trouton viscosity and change the draw mode, therefore, the critical draw ratio for draw resonance will change case by case even in isothermal and uniform tension melt spinning.

Since the cross-sectional area of the filament decreases monotonously with increasing distance from the spinneret, by differentiating with respect to the distance x rather than the cross-sectional area, one obtains another expression of draw modes as,

$$\frac{\partial^2 v}{\partial^2 x} \begin{cases} < 0 \\ = 0 \\ > 0 \end{cases} \tag{25}$$

Thus, the second order derivative of the filament velocity with respect to the distance x corresponds

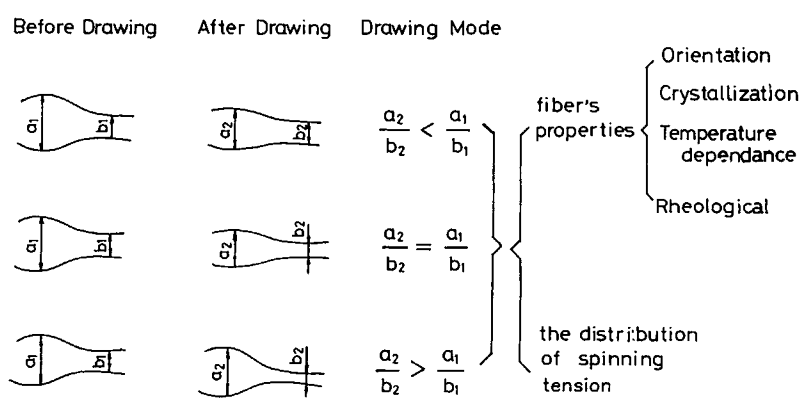

Figure 5 The promotive and suppressive factors affecting the draw resonance. Structure formation of the filament is an effective suppressive factor of draw resonance.

to the draw mode which is shown in eq. (4). By using eq. (25) one can evaluate qualitatively the draw mode from the velocity profiles of spinline, however, using eq. (25) instead of eq. (4) loses quantitative precision. In Figure 6, the velocity profiles in the spinline in isothermal melt spinning for power-law polymer fluids are shown. With the isothermal and uniform stress melt spinning the velocity profile of the spinline is represented by a straight line, and $\partial^2 v/\partial^2 x = 0$. This indicates an uniform draw mode. In isothermal and uniform tension melt spinning the velocity profile of the spinline has an "upward bend," and $\partial^2 v/\partial^2 x > 0$. The critical drawing ratio decreases with an increasing degree of upward bending.

A typical velocity profile of spinline in melt spinning[19] is shown in Figure 7. In this case the velocity profile can be separated into two regions, the unstable region ($\partial^2 v/\partial^2 x > 0$) and the stable region ($\partial^2 v/\partial^2 x < 0$). In the unstable region, the contributions of cooling, molecular orientation, and crystallization to the Trouton viscosity are small. Thus, the thinner the cross section of filament, the larger the extent the thinner section will be stretched to. In the stable region, however, the increasing effects of structure formation of the polymer fluid to the Trouton viscosity become so evident and even preponderant, that the thinner section of the filament is more inextensible than the thicker section. Hence, draw resonance is suppressed in this stable region. This indicates why draw resonance is rarely observed in real melt spinning.

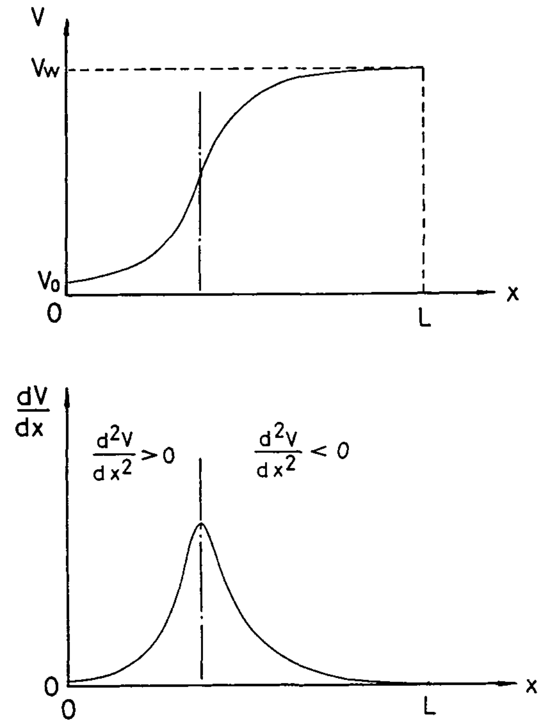

Figure 7 The velocity and velocity gradient profiles of the spinline in real melt spinning. In the region of $\partial^2 v/\partial^2 x > 0$, the filament has a nonuniformizing draw mode and in the region of $\partial^2 v/\partial^2 x < 0$, the filament has an uniformizing draw mode, which plays a preponderant contribution in suppressing draw resonance.

Fluctuation of Filament Diameter Along the Spinline

The developing manner of filament unevenness in melt spinning is the best evidence for the existence of the unstable and stable regions mentioned above. In the unstable region, the nonuniformizing draw mode acts to expand the filament unevenness. On the other hand, in the stable region, the uniformizing draw mode tends to diminish the filament unevenness. It is expected, therefore, that near the maximum point of the velocity gradient the coefficient of variation of the filament diameter reaches a maximum.

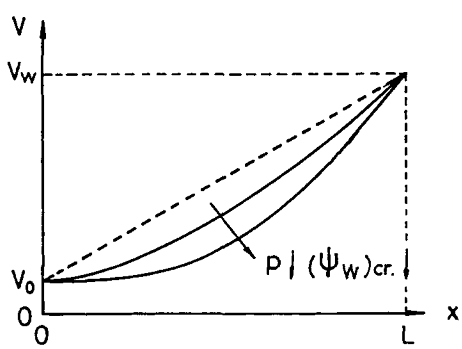

Figure 6 Velocity profile of the spinline in isothermal and uniform tension melt spinning for power-law polymer fluids. The critical draw ratio decreases with an increasing degree of upward bending of the velocity profile.

Melt spinning experiments were conducted to inspect the above deduction. Displayed in Table I are the spinning conditions. Filament diameters were measured on line by using an infrared ray type diameter monitor (Zimmer 460A/2) together with a micro computer system. The detailed description of

Table I Main Spinning Conditions

Polymer	Isopolypropylene
Molecular weight	135,000
Mass flow rate	2.50 g/min
Take-up velocity	500 m/min
Spinneret diameter	1 mm
Spinline length	330 cm

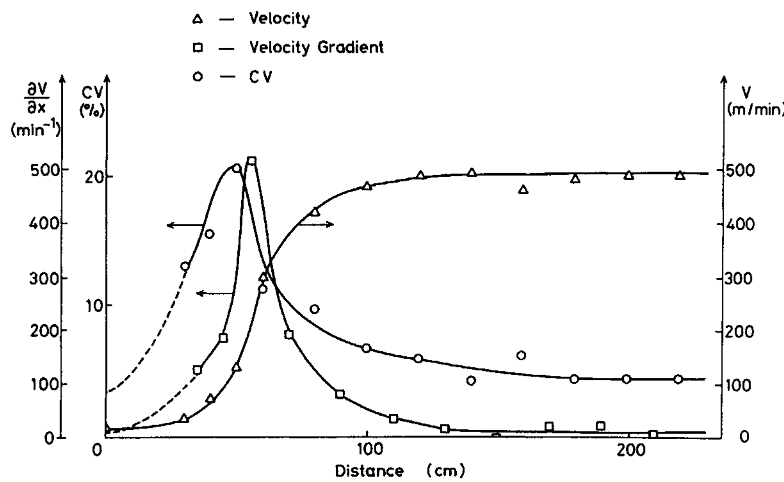

Figure 8 Experimental relationship between the velocity gradient of the filament and filament unevenness. Near the maximum point of the velocity gradient, CV% reaches a maximum.

the system is given in the cited literature.[20] The coefficients of variation (standard deviation divided by average) of the filament diameter were calculated from 8,000 data points. Figure 8 exemplifies the dynamically measured results. As we expected, the CV% of the filament diameter increases in the region of $(\partial^2 v/\partial^2 x > 0)$, reaches a maximum near the maximum point of velocity gradient $(\partial^2 v/\partial^2 x = 0)$, and then drops to a steady variation in the region of $(\partial^2 v/\partial^2 x < 0)$. Excellent consistency with the above deduction is observed. This, in turn, provides strong support to the proposed mechanism of draw resonance.

Here, it should be mentioned that, draw resonance is usually thought to be a cyclic variation phenomenon of filament, it seems different from filament unevenness. Ishihara and Kase[15] have demonstrated that draw resonance is a kind of limit cycle, which has a standing periodic oscillation. The distinct period and amplitude of the oscillation are thought to be characteristic of draw resonance. Their study, however, was restricted to a step response in the take-up velocity. If there is a disturbance source acting on the spinline, the amplitude and period of the oscillation will lose its distinct standing characteristic. Namely, if a new disturbance acts on the spinline during the transient state of the standing periodic oscillation corresponding to the last disturbance, an oscillation rather like unevenness with multiple frequencies and phase lags will be observed. Therefore, the filament unevenness expands and diminishes along the spinline.

CONCLUSIONS

Draw resonance in melt spinning can be closely connected with the draw mode of the filament. The local drawing ratio is expressed in eq. (2) and its cross-sectional area dependence is shown in eq. (3) and eq. (4). The nonuniformizing draw mode is the necessary condition of draw resonance and the mechanism of draw resonance in melt spinning lies in the nonuniformizing draw mode and the balance between the quantities of extrusion rate and take-up velocity. Analytical solutions show draw resonance is impossible in isothermal melt spinning under uniform stress for any power-law polymer fluid. The critical draw ratios for isothermal and uniform tension spinning of power-law polymer fluids, calculated through the draw mode method, are found to be in good agreement with those reported by Pearson and Shah[13] and Ishihara and Kase.[15] The proposition about the mechanism of draw resonance is supported by the agreements together with the success in analyzing the developing manner of filament unevenness along the spinline.

NOMENCLATURE

A	cross-sectional area of the filament
A_0	cross-sectional area of the filament at the spinneret
CV%	coefficient of variation of the filament diameter
F	spinning tension
L	spinline length

p	power-law index of the power-law fluids
S_p	stability parameter
t	time
T	temperature of the filament
T_0	temperature of the filament at the spinneret
v	velocity
V_0	velocity at the spinneret
V_w	velocity at the take-up bobbin
x	distance from the spinneret
β	the Trouton viscosity
β_0	a constant of the Trouton viscosity
ψ_w	draw ratio

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